## A- INTRODUCTION TO POINCARE CONJECTURE AND NON-LINEAR DIFFUSION

In mathematics, a metric, **g**, or distance function is a function which defines a distance between elements of a set.

For 2-dimensional surfaces without boundary, such as a beach ball, if every loop around the ball can be continuously tightened to a point, then the surface is topologically similar to a sphere.

The Poincare conjecture asserts that the same is true for 3-dimensional surfaces, or in a more rigorous terminology, The Poincare conjecture says that if a 3-dimensional manifold is compact, has no boundary and is simply connected, then it is homeomorphic to a 3-dimensional sphere. The concepts of "manifold", "compact", "no boundary", "simply connected", "homeomorphic" and "3-dimensional sphere". The details can be seen in:

http://en.wikipedia.org/wiki/Solution_of_the_Poincar%C3%A9_conjecture

Since 1906, a Russian mathematician Grigori Perelman attempted with a proof of the conjecture in three papers made available in 2002 and 2003 on arXiv.org. The proof followed the program of Richard Hamilton.

The Poincare conjecture was one of the most important open questions in topology. Clay Mathematics Institute offered a $1,000,000 prize for the solution. Perelman was awarded the Millennium Prize on 18 March 2010 and the prestigious Field Medals (equivalent to the Nobel Prize. He turned down both. He believes his contribution is an extension of the work on Ricci flows by U.S. mathematician Richard Hamilton, who first suggested a program for the solution.

It is possible to understand the mathematics leading to the solution of the Poincare Conjecture from a simple physical picture, the diffusion equation of heat flow. The following link expands this concept:

http://www.ias.ac.in/currsci/nov252006/1326.pdf

The diffusion coefficient K(x) is not a constant in general since the environment is usually heterogeneous. But when the region of the diffusion is approximately homogeneous, we can assume that

$K(x) = K$ and $\frac{\partial U}{\partial t} = K\Delta P + R(t,x,U)$

Where $\Delta U = \mathrm{div}(\nabla U)$ is the Laplacian operator, where the Laplacian is defined as

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

When there is no reaction occurs, the $\frac{\partial U}{\partial t} = K\Delta U$ equation is the diffusion equation: $\frac{\partial U}{\partial t} = K\Delta U$

The birth death rate of R in $\frac{\partial U}{\partial t} = K\Delta P + R(t,x,U)$ is the reaction term that incorporates the creation and destruction of the molecules that conducts heat.

In the Poincare proof, the Ricci flow resembles the heat the Ricci curvature becomes uniform as time progresses. However, there is an extra term Q (g, ∂ g) of lower order. Such a term is called the reaction term and equations of this form are known as reaction-diffusion equation. In order to understand such an equation, one needs to understand both the nature of the reaction term and conditions that govern whether the reaction or the diffusion terms dominate. The following link illustrates this concept in detail.

http://math.iisc.ernet.in/~gadgil/expos/poincare.pdf

B- DETAILED FORMALISM