Mechanics

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Mechanics

velocity

$$\bar{v} = \frac{\Delta s}{\Delta t}$$
$$\mathbf{v} = \frac{d\mathbf{s}}{dt}$$

acceleration

$$\bar{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t}$$
$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

equations of motion

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$\bar{v} = \tfrac{1}{2}(v + v_0)$$

newton's 2nd law

$$\sum \mathbf{F} = m\,\mathbf{a}$$
$$\sum \mathbf{F} = \frac{d\mathbf{p}}{dt}$$

weight

$$\mathbf{W} = m\,\mathbf{g}$$

dry friction

$$f \le \mu N$$

centrip. accel.

$$a_c = \frac{v^2}{r}$$
$$\mathbf{a}_c = -\,\omega^2\mathbf{r}$$

momentum

$$\mathbf{p} = m\mathbf{v}$$

impulse

$$\mathbf{J} = \bar{\mathbf{F}}\Delta t$$
$$\mathbf{J} = \int \mathbf{F}\,dt$$

impulse-momentum

$$\bar{\mathbf{F}}\Delta t = m\Delta\mathbf{v}$$
$$\int \mathbf{F}\,dt = \Delta\mathbf{p}$$

work

$$W = \bar{F}\Delta s \cos\theta$$
$$W = \int \mathbf{F}\cdot d\mathbf{s}$$

work-energy

$$\bar{F}\Delta s \cos\theta = \Delta E$$
$$\int \mathbf{F}\cdot d\mathbf{s} = \Delta E$$

kinetic energy

$$K = \tfrac{1}{2}mv^2$$

general p.e.

$$\Delta U = -\int \mathbf{F}\cdot d\mathbf{s}$$
$$\mathbf{F} = -\,\nabla U$$

gravitational p.e.

$$\Delta U_g = mg\Delta h$$

efficiency

$$\eta = \frac{W_{out}}{E_{in}}$$

power

$$\bar{P} = \frac{\Delta W}{\Delta t}$$
$$P = \frac{dW}{dt}$$
$$\bar{P} = \bar{F}v\cos\theta$$
$$P = \mathbf{F}\cdot\mathbf{v}$$

angular velocity

$$\bar{\boldsymbol{\omega}} = \frac{\Delta\boldsymbol{\theta}}{\Delta t}$$
$$\boldsymbol{\omega} = \frac{d\boldsymbol{\theta}}{dt}$$
$$\mathbf{v} = \boldsymbol{\omega}\times\mathbf{r}$$

angular acceleration

$$\bar{\boldsymbol{\alpha}} = \frac{\Delta\boldsymbol{\omega}}{\Delta t}$$
$$\boldsymbol{\alpha} = \frac{d\boldsymbol{\omega}}{dt}$$
$$\mathbf{a} = \boldsymbol{\alpha}\times\mathbf{r} - \omega^2\,\mathbf{r}$$

equations of rotation

$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$
$$\bar{\omega} = \tfrac{1}{2}(\omega + \omega_0)$$

2nd law for rotation

$$\sum \boldsymbol{\tau} = I\,\boldsymbol{\alpha}$$
$$\sum \boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$$

torque

$$\tau = rF\sin\theta$$
$$\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$$

moment of inertia

$$I = \sum mr^2$$
$$I = \int r^2\,dm$$

rotational work

$$W = \bar{\tau}\Delta\theta$$
$$W = \int \boldsymbol{\tau}\cdot d\boldsymbol{\theta}$$

rotational power

$$P = \tau\omega\cos\theta$$
$$P = \boldsymbol{\tau}\cdot\boldsymbol{\omega}$$

rotational k.e.

$$K = \tfrac{1}{2}I\omega^2$$

angular momentum

$$L = mrv\sin\theta$$
$$\mathbf{L} = \mathbf{r}\times\mathbf{p}$$
$$\mathbf{L} = I\,\boldsymbol{\omega}$$

universal gravitation

$$\mathbf{F}_g = -\frac{Gm_1m_2}{r^2}\,\hat{\mathbf{r}}$$

gravitational field

$$\mathbf{g} = -\frac{Gm}{r^2}\,\hat{\mathbf{r}}$$

gravitational p.e.

$$U_g = -\frac{Gm_1m_2}{r}$$

gravitational potential

$$V_g = -\frac{Gm}{r}$$

orbital speed

$$v = \sqrt{\frac{Gm}{r}}$$

escape speed

$$v = \sqrt{\frac{2Gm}{r}}$$

hooke's law

$$\mathbf{F} = -\,k\Delta\mathbf{x}$$

elastic p.e.

$$U_s = \tfrac{1}{2}k\Delta x^2$$

s.h.o.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

simple pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

frequency

$$f = \frac{1}{T}$$

angular frequency

$$\omega = 2\pi f$$

density

$$\rho = \frac{m}{V}$$

pressure

$$P = \frac{F}{A}$$

pressure in a fluid

$$P = P_0 + \rho gh$$

buoyancy

$$B = \rho g V_{displaced}$$

mass flow rate

$$I = \frac{m}{t}$$

volume flow rate

$$\varphi = \frac{V}{t}$$

mass continuity

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

volume continuity

$$A_1 v_1 = A_2 v_2$$

bernoulli's equation

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

dynamic viscosity

$$\eta = \frac{\bar{F}/A}{\Delta v_x/\Delta z}$$
$$\eta = \frac{F/A}{dv_x/dz}$$

kinematic viscosity

$$\nu = \frac{\eta}{\rho}$$

aerodynamic drag

$$R = \tfrac{1}{2}\rho C A v^2$$

mach number

$$Ma = \frac{v}{c}$$

reynolds number

$$Re = \frac{\rho v D}{\eta}$$

froude number

$$Fr = \frac{v}{\sqrt{g\ell}}$$

young's modulus

$$\frac{F}{A} = E\frac{\Delta\ell}{\ell_0}$$

shear modulus

$$\frac{F}{A} = G\frac{\Delta x}{y}$$

bulk modulus

$$\frac{F}{A} = K\frac{\Delta V}{V_0}$$

surface tension

$$\gamma = \frac{F}{\ell}$$

Thermal Physics

solid expansion

$$\Delta\ell = \alpha\ell_0\Delta T$$
$$\Delta A = 2\alpha A_0\Delta T$$
$$\Delta V = 3\alpha V_0\Delta T$$

liquid expansion

$$\Delta V = \beta V_0\Delta T$$

sensible heat

$$Q = mc\Delta T$$

latent heat

$$Q = mL$$

ideal gas law

$$PV = nRT$$

molecular constants

$$nR = Nk$$

maxwell-boltzmann

$$p(v) = \frac{4v^2}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}}$$

molecular k.e.

$$\langle K\rangle = \frac{3}{2}kT$$

molecular speeds

$$v_p = \sqrt{\frac{2kT}{m}}$$
$$\langle v\rangle = \sqrt{8kT\pi m}$$
$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

heat flow rate

$$\bar{\Phi} = \frac{\Delta Q}{\Delta t}$$
$$\Phi = \frac{dQ}{dt}$$

thermal conduction

$$\Phi = \frac{kA\Delta T}{\ell}$$

stefan-boltzmann law

$$\Phi = \varepsilon\sigma A(T^4 - T_0^4)$$

wien displacement law

$$\lambda_{max} = \frac{b}{T}$$

internal energy

$$\Delta U = \tfrac{3}{2}nR\Delta T$$

thermodynamic work

$$W = -\int P\,dV$$

1st law of thermo.

$$\Delta U = Q + W$$

entropy

$$\Delta S = \frac{\Delta Q}{T}$$
$$S = k\log w$$

efficiency

$$\mathcal{E}_{real} = 1 - \frac{Q_C}{Q_H}$$
$$\mathcal{E}_{ideal} = 1 - \frac{T_C}{T_H}$$

c.o.p.

$$COP_{real} = \frac{Q_C}{Q_H - Q_C}$$
$$COP_{ideal} = \frac{T_C}{T_H - T_C}$$

Waves & Optics

periodic waves

$$v = f\lambda$$

frequency

$$f = \frac{1}{T}$$

beat frequency

$$f_{beat} = f_{high} - f_{low}$$

intensity

$$I = \frac{\langle P\rangle}{A}$$

intensity level

$$L_I = 10\log\left(\frac{I}{I_0}\right)$$

pressure level

$$L_P = 20\log\left(\frac{\Delta P_{max}}{\Delta P_0}\right)$$

interference fringes

$$n\lambda = d\sin\theta$$
$$\frac{n\lambda}{d} \approx \frac{x}{L}$$

index of refraction

$$n = \frac{c}{v}$$

snell's law

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

critical angle

$$\sin\theta_c = \frac{n_2}{n_1}$$

image location

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

image size

$$M = \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

spherical mirrors

$$f \approx \frac{r}{2}$$

Electricity & Magnetism

coulomb's law

$$F = k\frac{q_1q_2}{r^2}$$

electric field, def.

$$\mathbf{E} = \frac{\mathbf{F}_E}{q}$$

electric field, around charges

$$\mathbf{E} = k\sum\frac{q}{r^2}\,\hat{\mathbf{r}}$$
$$\mathbf{E} = k\int\frac{dq}{r^2}\,\hat{\mathbf{r}}$$

field and potential

$$\bar{E} = -\frac{\Delta V}{d}$$
$$\mathbf{E} = -\,\nabla V$$

electric potential, def.

$$\Delta V = \frac{\Delta U_E}{q}$$

electric potential, around charges

$$V = k\sum\frac{q}{r}$$
$$V = k\int\frac{dq}{r}$$

capacitance

$$C = \frac{Q}{V}$$

plate capacitor

$$C = \frac{\kappa\varepsilon_0 A}{d}$$

cylindrical capacitor

$$C = \frac{2\pi\kappa\varepsilon_0\ell}{\ln(b/a)}$$

spherical capacitor

$$C = \frac{4\pi\kappa\varepsilon_0}{(1/a) - (1/b)}$$

capacitive p.e.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

electric current

$$\bar{I} = \frac{\Delta q}{\Delta t}$$
$$I = \frac{dq}{dt}$$

ohm's law

$$V = IR$$
$$\mathbf{E} = \rho\,\mathbf{J}$$
$$\mathbf{J} = \sigma\mathbf{E}$$

resitivity-conductivity

$$\rho = \frac{1}{\sigma}$$

electric resistance

$$R = \frac{\rho\ell}{A}$$

electric power

$$P = VI = I^2R = \frac{V^2}{R}$$

resistors in series

$$R_s = \sum R_i$$

resistors in parallel

$$\frac{1}{R_p} = \sum\frac{1}{R_i}$$

capacitors in series

$$\frac{1}{C_s} = \sum\frac{1}{C_i}$$

capacitors in parallel

$$C_p = \sum C_i$$

magnetic force, charge

$$F_B = qvB\sin\theta$$
$$\mathbf{F}_B = q\,\mathbf{v}\times\mathbf{B}$$

magnetic force, current

$$F_B = I\ell B\sin\theta$$
$$d\mathbf{F}_B = I\,d\boldsymbol{\ell}\times\mathbf{B}$$

biot-savart law

$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

solenoid

$$B = \mu_0 nI$$

straight wire

$$B = \frac{\mu_0 I}{2\pi r}$$

parallel wires

$$\frac{F_B}{\ell} = \frac{\mu_0}{2\pi}\frac{I_1I_2}{r}$$

electric flux

$$\Phi_E = EA\cos\theta$$
$$\Phi_E = \int \mathbf{E}\cdot d\mathbf{A}$$

magnetic flux

$$\Phi_B = BA\cos\theta$$
$$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A}$$

motional emf

$$\mathcal{E} = B\ell v$$

induced emf

$$\bar{\mathcal{E}} = -\frac{\Delta\Phi_B}{\Delta t}$$
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

gauss's law

$$\oiint \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$
$$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$$

no one's law

$$\oiint \mathbf{B}\cdot d\mathbf{A} = 0$$
$$\nabla\cdot\mathbf{B} = 0$$

faraday's law

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$
$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

ampere's law

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} + \mu_0 I$$
$$\nabla\times\mathbf{B} = \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t} + \mu_0\,\mathbf{J}$$

Modern Physics

time dilation

$$t' = \frac{t}{\sqrt{(1 - v^2/c^2)}}$$

length contraction

$$\ell' = \ell\sqrt{(1 - v^2/c^2)}$$

relativistic mass

$$m' = \frac{m}{\sqrt{(1 - v^2/c^2)}}$$

relative velocity

$$u' = \frac{u + v}{1 + uv/c^2}$$

relativistic energy

$$E = \frac{mc^2}{\sqrt{(1 - v^2/c^2)}}$$

relativistic momentum

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{(1 - v^2/c^2)}}$$

energy-momentum

$$E^2 = p^2c^2 + m_0^2c^4$$

mass-energy

$$E = mc^2$$

relativistic doppler effect

$$\frac{\lambda}{\lambda_0} = \frac{f_0}{f} = \sqrt{\left(\frac{1 + v/c}{1 - v/c}\right)}$$

photon energy

$$E = hf$$

photoelectric effect

$$K_{max} = E - \phi = h(f - f_0)$$

photon momentum

$$p = \frac{h}{\lambda}$$

schroedinger's equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t)$$
$$E\psi(\mathbf{r}) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r})$$

uncertainty principle

$$\Delta p_x\,\Delta x \ge \frac{\hbar}{2}$$
$$\Delta E\,\Delta t \ge \frac{\hbar}{2}$$

rydberg equation

$$\frac{1}{\lambda} = -R_\infty\left(\frac{1}{n^2} - \frac{1}{n_0^2}\right)$$